ON GENERIC INDEPENDENT FAMILIES

COREY BACAL SWITZER

A family $\mathcal{I} \subseteq [\omega]^\omega$ is called *independent* if for every two finite, disjoint $\mathcal{A}, \mathcal{B} \subseteq \mathcal{I}$ the set $\bigcap \mathcal{A} \setminus \bigcup \mathcal{B}$ is infinite. The *independence number*, $\mathfrak{i}$, is the least size of a $\subseteq$-maximal independent family. Despite being one of the classical cardinal characteristics introduced in the 30's, many basic questions remain open about $\mathfrak{i}$. For instance, it is unknown if consistently $\mathfrak{i}$ can be less than the almost disjointness number or have countable cofinality.

Part of the issue stems from the fact that very little is known about what varieties of independent families can exist. We will survey several results aimed at rectifying this. Central to these is that the standard way to build a maximal independent family of desired size by forcing actually introduces a *selective* one: that is, a strongly forcing indestructible independent family with Ramsey theoretic properties. As a consequence we show that under $\mathfrak{p} = \mathfrak{c}$ there are a selective independent families and moreover, that there may be a selective independent family of any desired uncountably cofinal size. These results rely heavily on combinatorics of filters associated with independent families which we plan to discuss as well.

This is work with Vera Fischer.

(C. B. Switzer) Institut für Mathematik, Kurt Gödel Research Center, Universität Wien, Kolingasse 14-16, 1090 Wien, AUSTRIA

Email address: corey.bacal.switzer@univie.ac.at

Acknowledgments: The author would like to thank the Austrian Science Fund (FWF) for the generous support through grant number ESP 548.